Short Papers

A Simplified Formulation to Analyze Inhomogeneous Waveguides With Lossy Chiral Media Using the Finite-Element Method

Luis Valor and Juan Zapata

***Abstract*— In this paper, an efficient finite-element formulation is presented for the analysis of the propagation characteristics in arbitrarily shaped lossy inhomogeneous waveguides loaded with chiral media. It is a simplified form of the one proposed in [1] for the bi-anisotropic media. In this formulation, showing no spurious modes, the frequency or the propagation constants may be treated as eigenvalues of a resulting sparse quadratic eigenproblem. However, in order to handle losses easily and to facilitate computation of complex modes, the frequency is specified as an input parameter and the eigensystem is solved for the complex propagation constant as the eigenvalue. This sparse eigensystem is further transformed into a generalized one, thus maintaining the sparse properties of the matrices. New numerical finite-element results are presented.**

***Index Terms*—Chiral medium, finite element, sparse eigensystem.**

I. Introduction

A chiral medium is a particular case of bi-isotropic medium, characterized by linear constitutive relations which couple the electric and magnetic field by three scalars [2]. Besides the potential applications in optical and suboptical frequencies, considerable interest has been generated in isotropic chiral media. This interest is based on the existence of one additional parameter, the chirality admittance ξ_c, that could make the practical designs more flexible.

In this paper, in order to obtain propagation constants and fields in a chirowaveguide, we propose a method based on the finite-element method (FEM), which has no limitation concerning the cross-section shape of the waveguides. This cross section is divided into triangular hybrid vector elements as proposed in [3], which permit to solve waveguides with reentrant corners and to eliminate the possibility of the appearance of spurious modes.

The first formulation to solve chirowaveguides using the FEM was proposed by Svedin [4]. It employed 6 degrees of freedom per point. The method proposed in this paper is formulated in terms of the magnetic field and only one degree of freedom per point is used [3].

The discretization process leads to solve a sparse eigenvalue problem in which the frequency or the propagation constant can be selected as an input parameter and the other one as an eigenvalue. However, when losses or complex modes are present in a waveguide, the propagation constant becomes complex: $\gamma = \alpha + j\beta$. Thus, in order to solve the eigensystem, it is preferable to sweep the positive real frequency axis by taking the frequency as an input parameter, and obtaining γ as the eigenvalue, instead of sweeping the whole complex γ plane by fixing α (or β) and iterating on β (or α) to search for a complex frequency with a negligible imaginary part as an eigenvalue.

In this paper, the propagation constant of coupled slot lines and coupled microstrips chirowaveguides, together with the analysis versus chirality admittance, are presented.

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L. Valor is with the Departamento de Electrónica y Teoría de Circuitos, E.T.S.I. Telecomunicación, Universidad Europea de Madrid, 28670 Madrid, Spain.

J. Zapata is with the Departamento de Electromagnetismo y Teoría de Circuitos, Universidad Politécnica de Madrid, E.T.S.I. Telecomunicación, 28040 Madrid, Spain.

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II. Theory

A chiral medium is characterized by its constitutive relations. In this paper, the Sihvola–Lindell relations are adopted:

$$\begin{aligned}\vec{D} &= \varepsilon\vec{\mathbf{E}} - \xi\vec{\mathbf{H}}\\ \vec{B} &= \xi\vec{\mathbf{E}} + \mu\vec{\mathbf{H}}\end{aligned} \tag{1}$$

where $\vec{D}$ is the electric flux density, $\vec{\mathbf{E}}$ the electric field, $\vec{B}$ the magnetic flux density, and $\vec{\mathbf{H}}$ the magnetic field. The parameters $\varepsilon = \varepsilon_o\varepsilon_r$, $\mu = \mu_o\mu_r$ are the permittivity and permeability, respectively, $\xi/j\sqrt{\varepsilon_o\mu_o} = \mu\xi_c/\sqrt{\varepsilon_o\mu_o}$ the Pasteur parameter, and ξ_c the chirality admittance.

We consider a lossy waveguide with translation symmetry, arbitrarily shaped cross-section Ω in the x–y plane, inhomogeneously filled with chiral media, and bounded by electric (Γ_1) and magnetic (Γ_2) walls.

From source-free Maxwell equations in the frequency domain and assuming that the electromagnetic field in the waveguide varies as $e^{(j\omega t-\gamma z)}$, we can obtain the Helmhotz equation for the magnetic field as

$$\frac{1}{\varepsilon_r}\nabla\times\nabla\times\vec{\mathbf{H}} + \frac{2}{\varepsilon_r}j\omega\xi\nabla\times\vec{\mathbf{H}} - \frac{1}{\varepsilon_r}\omega^2\xi^2\vec{\mathbf{H}} - k_o^2\mu_r\vec{\mathbf{H}} = 0 \tag{2}$$

with the boundary conditions termed as

$$\hat{n}\times(\varepsilon^{-1}\nabla\times\vec{\mathbf{H}} - j\omega\varepsilon^{-1}\xi\vec{\mathbf{H}}) = 0 \qquad \text{on } \Gamma_1 \tag{3}$$

$$\hat{n}\times\vec{\mathbf{H}} = 0 \qquad \text{on } \Gamma_2 \tag{4}$$

where ω is the angular frequency, $\gamma = \alpha + j\beta$ the complex propagation constant, and k_o the free-space wavenumber.

Expressions (2)–(4) coincide with those proposed in [1] specified for chiral media. By replacing

$$\varepsilon_r = \varepsilon_r' + \frac{\mu\xi_c^2}{\varepsilon_o} \tag{5}$$

a new simplified formulation is obtained:

$$\nabla\times\nabla\times\vec{\mathbf{H}} - 2\omega\mu\xi_c\nabla\times\vec{\mathbf{H}} - k_o^2\varepsilon_r'\vec{\mathbf{H}} = 0 \tag{6}$$

This expression can also be obtained from Post–Jaggard relations.

Applying the Galerkin method to (6), splitting the trial functions $\vec{H}$, the test functions $\vec{w}$, and the operator ∇ into their transverse and axial parts, and using several vectorial identities, we obtain the following expression:

$$\begin{aligned}\frac{1}{\varepsilon_r'}\int_\Omega\Big\{&(\nabla_t\times\vec{w}_t)(\nabla_t\times\vec{H}_t) + \nabla_t w_z\cdot[\nabla_t H_z + \gamma\hat{H}_t]\\ &+ \vec{w}_t\big[\big(-\gamma + 2\omega\mu_r\xi_c\varepsilon_r'\big)\nabla_t H_z\\ &\qquad - \big(\gamma^2 - 2\omega\gamma\mu\xi_c\varepsilon_r'\bar{a} + k_o^2\mu_r\big)\vec{H}_t\big]\\ &+ w_z\hat{z}\big[-2\omega\mu_r\varepsilon_r'\mu_r\xi_c\nabla_t\times H_t - k_o^2\bar{\mu}\varepsilon_r'\hat{z}H_z\big]\Big\}\,d\Omega = 0\end{aligned} \tag{7}$$

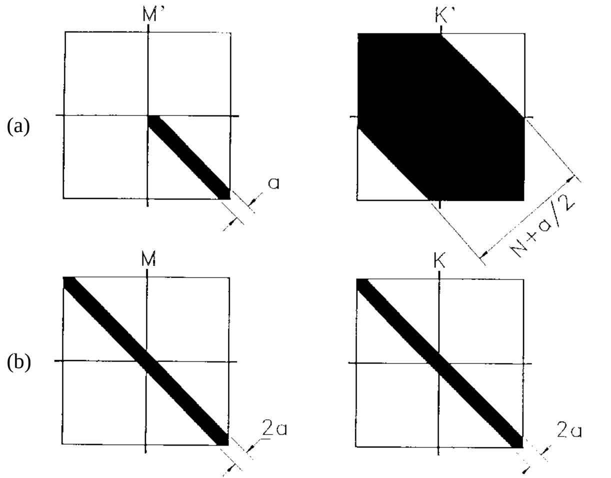

Fig. 1. Matrices of the generalized eigensystem (a) before and (b) after the reordering of the matrices.

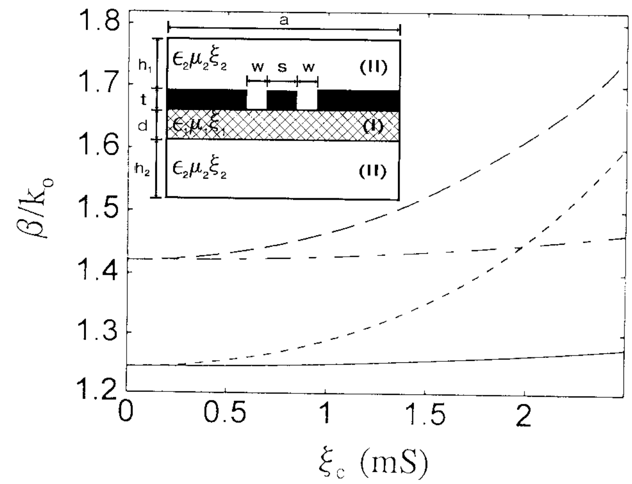

Fig. 2. Normalized propagation constant versus chirality admittance for two coupled slotlines chirowaveguide. $a = 1.55$ mm, $h_1 + t = h_2 = 1.44$ mm, $t = 50$ μm, $w = s = 0.2$ mm, $d = 0.22$ mm, $\varepsilon_1 = 3.75$, $\varepsilon_2 = 1$, $\mu_1 = \mu_2 = 1$, $f = 90$ GHz. (— – —) even mode with $\xi_c = \xi_1$ and $\xi_2 = 0$, (—) odd mode with $\xi_c = \xi_1$ and $\xi_2 = 0$, (– –) even mode with $\xi_c = \xi_1 = \xi_2$, (- - - -) odd mode with $\xi_c = \xi_1 = \xi_2$.

where we have imposed homogeneous boundary conditions (4) on magnetic walls and natural boundary conditions (3) on electric walls.

By discretizing the expression in (7) by hybrid finite elements as described in [3], we can obtain an eigenvalue problem of the form

$$\{H\}(\gamma^2[A] + \gamma[B] + [C] + \gamma\omega[D] + \omega[E] + \omega^2[F]) = 0. \tag{8}$$

The eigensystem is quadratic if both the input parameter is the propagation constant and the eigenvalue is the frequency and vice versa. However, for the reasons explained above, the frequency is taken as the input parameter and the eigensystem is solved for the complex propagation constant as the eigenvalue. This leads to a quadratic eigenvalue problem expressed as

$$(\gamma^2[M_1] + \gamma[M_2] + [M_3])\{H\} = 0 \tag{9}$$

where

$$\begin{aligned}
[M_1] &= \sum_e \begin{bmatrix} [T_3] & [0] \\ [0] & [0] \end{bmatrix} \\
[M_2] &= \sum_e \begin{bmatrix} \omega[T_9] & [T_4] \\ [T_2] & [0] \end{bmatrix} \\
[M_3] &= \sum_e \begin{bmatrix} [T_1] - \omega^2[T_6] & -\omega[T_8] \\ -\omega[T_{10}] & [T_5] - \omega^2[T_7] \end{bmatrix}
\end{aligned} \tag{10}$$

where the submatrices $[T_i]$ are given in the Appendix. It must be pointed out that the matrices in (9) are sparse and, in a general lossy case, complex, non-Hermitian, and asymmetrical. Since there is no public subroutine to directly solve this type of eigensystem, the sparse quadratic eigensystem (9) is transformed into a generalized one:

$$[K]'\{X\}' - \gamma[M]'\{X\}' = 0 \tag{11}$$

with double-dimension wide-band matrices, by setting

$$\begin{aligned}
[K]' &= \begin{bmatrix} [0] & [I] \\ [M_3] & [M_2] \end{bmatrix} \\
[M]' &= \begin{bmatrix} [I] & [0] \\ [0] & -[M_1] \end{bmatrix} \\
\{X\}' &= \begin{Bmatrix} \{H\} \\ \{\bar{H}\} \end{Bmatrix}
\end{aligned} \tag{12}$$

where $\{\bar{H}\}$ is an unknown vector. In Fig. 1(a), the terms that would be necessary to store in order to solve this system are marked in black. We can observe that the band of the matrix $[K]'$ is very large. In order to obtain a more efficient generalized eigenvalue problem, with narrower band matrix, the terms of the matrices are reordered, as described in [5], obtaining sparse matrices, as shown in Fig. 1(b). The final reordered generalized eigensystem

$$[K]\{X\} - \gamma[M]\{X\} = 0 \tag{13}$$

with sparse narrow-band singular matrices is solved by the subspace iteration algorithm.

The formulation proposed in this paper is substantially simpler than the equivalent expression that can be derived from the integral form [1, eq. (10)] when this is particularized for chiral media. It allows an easier implementation and a shorter computing time. In the particular case where the chirality parameter vanishes, this formulation is then the same as the one proposed in [6] for isotropic media.

III. Numerical Examples

The proposed formulation has been validated by analyzing a circular chirowaveguide presented in [4, Fig. 4], a rectangular chirowaveguide from [7, Fig. 3], and a partially loaded circular chirowaveguide from [8, Fig. 2]. In all the cases, our results coincided greatly with those of [4], [7], and [8].

Other chirowaveguides such as coupled slotlines or coupled microstrip on chiral ridges have been analyzed and the results are presented in Figs. 2 and 3. In Fig. 2, two coupled slotlines chirowaveguides are shown. This figure presents the normalized phase constant as a function of ξ_c for both even and odd modes. For $\xi_1 = \xi_2 = \xi_c = 0$ mS, our results coincide with those obtained in [9] and [11]. It can be seen that the phase constant increases slightly with increasing $\xi_c = \xi_1$ and $\xi_2 = 0$. However, this effect is stronger when there is chirality in regions I and II simultaneously, $\xi_1 = \xi_2 = \xi_c$.

Analysis has also been performed for two coupled microstrips on chiral ridges (Fig. 3). In this case, the phase constant also increases with ξ_c being stronger if there is chirality in Regions I and II simultaneously, $\xi_1 = \xi_2 = \xi_c$, but this effect is smoother than in the example above because the permittivity is bigger. In this example, the results for $\xi_1 = \xi_2 = \xi_c = 0$ mS have a good agreement with the ones presented in [10].

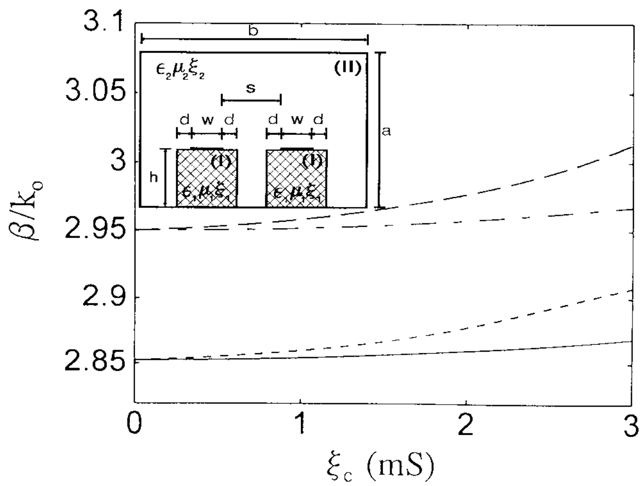

Fig. 3. Normalized propagation constant versus chirality admittance for a coupled microstrips on chiral ridges $w = 0.1$ mm, $h = 0.1$ mm, $d = 0.05$, $a = 1.3$ mm, $b = 2.51$ mm, $s = .3$ mm, $\varepsilon_1 = 12.85$, $\varepsilon_2 = 1$, $\mu_1 = \mu_2 = 1$, $f = 90$ GHz. (— — —) even mode with $\xi_c = \xi_1$ and $\xi_2 = 0$, (—) odd mode with $\xi_c = \xi_1$ and $\xi_2 = 0$, (– –) even mode with $\xi_c = \xi_1 = \xi_2$, (- - - -) odd mode with $\xi_c = \xi_1 = \xi_2$.

IV. Conclusion

In this paper, a simplified finite-element formulation for solving arbitrarily shaped waveguides including lossy inhomogeneous chiral media has been proposed. It is a simplified form of the one proposed in [1] for the bi-anisotropic media. Spurious-mode suppression is also achieved. In order to facilitate the analysis of waveguides, which can support complex modes and/or have losses, the frequency is introduced as the input parameter to obtain the complex propagation constant as the result. The formulation leads to a quadratic eigenvalue problem, which is transformed into a double-dimension generalized one. After a reordering, the eigensystem is solved by the subspace method, taking full advantage of the sparsity of the matrices. The proposed method has been validated by analyzing various chirowaveguides. Some new results have also been presented.

Appendix

In this paper, the submatrices $[T_i]$ are given by

$$[T_1] = \iint_e [A]^T [A]\, dp\, dq$$

$$[T_2] = \iint_e [D]^T [T]\, dp\, dq$$

$$[T_3] = \iint_e [T]^T [T]\, dp\, dq$$

$$[T_4] = \iint_e [T]^T [D]\, dp\, dq$$

$$[T_5] = \iint_e [D]^T [D]\, dp\, dq$$

$$[T_6] = \iint_e \mu\varepsilon_o [T]^T [T]\, dp\, dq$$

$$[T_7] = \iint_e \mu\varepsilon_o [N]^T [N]\, dp\, dq$$

$$[T_8] = \iint_e 2\mu_r\xi_c [T]^T [\bar{a}][D]\, dp\, dq$$

$$[T_9] = \iint_e 2\mu_r\xi_c [T]^T [\bar{a}][T]\, dp\, dq$$

$$[T_{10}] = \iint_e 2\mu_r\xi_c [N]^T [A]\, dp\, dq$$

where

$$[\bar{a}] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad [T] = \begin{bmatrix} \langle T_p \rangle \\ \langle T_q \rangle \end{bmatrix} \quad [N] = \langle N_i \rangle$$

$$[A] = \left\langle \frac{\partial T_q}{\partial p} - \frac{\partial T_p}{\partial q} \right\rangle \quad [D] = \begin{bmatrix} \left\langle \frac{\partial N_i}{\partial p} \right\rangle \\ \left\langle \frac{\partial N_i}{\partial q} \right\rangle \end{bmatrix}.$$

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